Homework Assignment 2

Biomedical Signal and Image Processing, Spring, 2010

4800-697:002

Due February 3

1. What signal-to-noise ratio (in decibels) would have the norm of the signal as exactly half the norm of the noise? Give an explanation of your answer. Also, what would be the actual ratio, $||x||/||n||$, if the value of the associated SNR is -12 dB?

2. Suppose s is a discrete signal of length $N = 128$ with entries given by $s[k] = 109 \cdot \sin(2\pi(0.415)k)/(2\pi k)$ for $k = 1, \ldots, 128$

   (a) Using a computer, compute the root mean square norm of s.

   (b) If noise n of root mean square norm $||n|| = 0.08$ is added to the signal s so that $x = s + n$, what is the SNR in x?

3. The continuous *sinc* function is an important function because of its simple Fourier transform. Define the function by the formula

$$sinc(t) = sin(\pi t)/(\pi t)$$

   if $t \neq 0$, and $sinc(0) = 1$. The digital *sinc* function is obtained, as usual, by sampling the continuous function at uniformly spaced intervals.

   Using a computer plotter, such as Matlab, make plots of the digital version of the *sinc* function over the interval $-3 \leq t \leq 3$ for sampling interval $T = 0.1$ **and** the real value of its corresponding digital Fourier transform for each of the following cases:

   (a) sinc(x)

   (b) sinc(3x)

   (c) sinc(x/3)

4. Using ImageJ, input the medical image from the samples files given there labeled 'CT.' Then determine the size of the image as a matrix, provide a print of a histogram of the image, provide a printout of the image, and a printout of the surface plot of the image.

5. In the following, suppose the Discrete Fourier Transform is applied to a signal $x[n]$ that has period $N = 100$. For each of the following, a signal $x[n]$ is given. Sketch the DFT of x in each case over the interval $0 \leq k \leq 99$, labeling important points on the frequency axis.

   (a) $x[n] = \delta[n-1]$

   (b) $x[n] = \exp(-12\pi n/100)$.

   (c) $x[n] = \cos(16\pi n/100)$

6. This exercise is about *aliasing.* Suppose continuous sinusoidal signal $x_1(t) = \cos(2\pi t)$ is sampled at integer multiples of $T = 0.25$. Determine the formula of a different sinusoidal signal $x_2(t)$ with a higher actual frequency but which has the *exact same* samples as $x_1(t)$ at those points. Show that your $x_2(t)$ signal does indeed have the same samples.