This problem set covers the material in the first two sections of the course. The Excel file *Data for problem set 1.xls* can be downloaded from the webpage.

1. Answer the following using the U.S. data contained on the Excel file *Data for problem set 1.xls*
   a. In Excel, calculate the U.S. annual growth rate of real GDP from 1948.2 to 2000.4
   b. The National Bureau of Economic Research (NBER) defines a recession as three consecutive quarters of negative growth. When was the last recession?
   c. In Excel, plot the Okun relationship for 1980.1 to 2000.4

2. The Cobb-Douglas production function has a long tradition in economics. See the appendix in chapter 3. The Cobb-Douglas production function assumes that output, capital and labor are linked by the following equation: \( Y = K^\alpha L^{1-\alpha} \) where \( 0 < \alpha < 1 \).
   a. Prove that the Cobb-Douglas production function exhibits constant returns to scale in both inputs. Make sure you show all your work.
   b. Prove that the Cobb-Douglas production function exhibits decreasing returns to scale in capital. Make sure you show all your work.
   c. Prove that the Cobb-Douglas production function exhibits decreasing returns to scale in labor. Make sure you show all your work.
   d. Show that the sum of the payments to labor and payments to capital equal total income \( Y \). Make sure you show all your work.
   e. According to the Cobb-Douglas production function, the ratio of labor income to total income equals \((1- \alpha)\). What value of the output elasticity \( \alpha \) is consistent with U.S. data?

3. Republicans have been advocating reductions in the capital gains tax rate for years. The capital gains tax rate is the tax rate applied to increases in stock prices (or capital gains). The argument put forward is that if people retain a higher percentage of their capital gains then private saving (\( S_{pr} \)) and the supply of capital would increase. The following asks you to use the long-run, static model of section II to trace through the effects of a capital gains tax cut. Assume that the economy is closed so that \( NX = 0 \).
   a. Using the four diagrams of the production function and the factor markets, show the effects of a capital gains tax cut on the quantity of labor (\( \bar{L} \)), the real wage rate (\( W/P \)), the quantity of capital (\( \bar{K} \)) and the real rental rate (\( R/P \)).
   b. Using the closed-economy loanable funds market, show the effects of a capital gains tax cut on the real interest rate (\( r \)) and real investment (\( I \)) in the long-run.
c. Using the quantity theory of money, show the effects of a capital gains tax cut on the price level \( P \) in the long-run.

d. Could Alan Greenspan, chairman of the Federal Reserve, increase the long-run value of output even more by printing money? Explain why or why not.

4. Suppose that the U.S. economy is a small, open-economy. The following asks you to use the long-run, open-economy model of chapter 8 to trace through the effects of a capital gains tax cut.

a. Using the open-economy loanable funds market, show the effects of a capital gains tax cut on the real interest rate \( r \), real investment \( I \) and real net exports \( NX \) in the long-run.

b. Using the diagram that determines the real exchange rate, show the effects of a capital gains tax cut on the real exchange rate \( e \) and real net exports \( NX \) in the long-run.

5. Researchers have shown that a Solow growth model with a Cobb-Douglas production function of \( Y = K^{0.3} L^{0.7} \) can describe cross-country growth differences.

a. What is the per-worker production function, \( y = f(k) \)?

b. Assuming no technological progress, find the steady-state capital stock per worker \( k^* \), output per worker \( y^* \) and consumption per worker \( c^* \) in terms of the saving rate \( s \), the depreciation rate \( \bar{d} \) and the population growth rate \( n \).

c. Suppose that two countries have the same steady-state capital stock per worker \( k^* \) but one country has a high initial capital stock per worker \( k_0^{high} \) and the other has a low initial capital stock per worker \( k_0^{low} \). Explain which of the two countries would experience faster transitional growth. You may use the diagram of the Solow growth model to aid in your discussion.

d. Let \( n + \bar{d} = 0.07 \) Solve for the Golden Rule level of capital per worker \( k^{gold} \).

e. What saving rate would generate such a Golden Rule level of capital per worker? Why may it be difficult for policymakers to encourage people to save at that rate?