This problem set covers the material in the first two sections of the course.

1. Answer the following using the U.S. data contained on the Excel file Data for problem set 1.xls
   a. See enclosed spreadsheet Data key for problem set 1.xls
   b. The last recession occurred in the first quarter of 1991.
   c. See enclosed spreadsheet Data key for problem set 1.xls

2. The Cobb-Douglas production function has a long tradition in economics. See the appendix in chapter 3. The Cobb-Douglas production function assumes that output, capital and labor are linked by the following equation: \( Y = K^\alpha L^{1-\alpha} \) where \( 0 < \alpha < 1 \).
   a. To prove constant returns to scale in both inputs, you must show that \( \frac{cY}{cK} = \frac{cL}{cL} = 1 \).
   b. To prove decreasing returns to scale in capital, you must show that the first derivative with respect to capital is positive and the second derivative is negative.
      \[
      \frac{\partial F}{\partial K} = \alpha K^{\alpha-1} L^{-\alpha} = \alpha \left( \frac{L}{K} \right)^{1-\alpha} > 0 \quad \text{since} \quad \alpha > 0 \\
      \frac{\partial^2 F}{\partial K^2} = \alpha(\alpha - 1) K^{\alpha-2} L^{-\alpha} > 0 \quad \text{since} \quad 0 < \alpha < 1
      \]
   c. To prove decreasing returns to scale in labor, you must show that the first derivative with respect to labor is positive and the second derivative is negative.
      \[
      \frac{\partial F}{\partial L} = (1 - \alpha) K^{\alpha} L^{-\alpha} = (1 - \alpha) \left( \frac{K}{L} \right)^{\alpha} > 0 \quad \text{since} \quad \alpha < 1 \\
      \frac{\partial^2 F}{\partial L^2} = (1 - \alpha)(-\alpha) K^{\alpha-1} L^{-\alpha-1} > 0 \quad \text{since} \quad 0 < \alpha < 1
      \]
   d. Show that the sum of the payments to labor and payments to capital equal total income.
      \[
      L \cdot MPL + K \cdot MPK = L(1 - \alpha) K^{\alpha} L^{-\alpha} + K \alpha K^{\alpha-1} L^{1-\alpha} = K^{\alpha} L^{1-\alpha} - \alpha K^{\alpha-1} L^{1-\alpha} + \alpha K^{\alpha} L^{1-\alpha}
      \]
   e. A value of 0.3 for the output elasticity \( \alpha \) is consistent with U.S. data.
3. Republicans have been advocating reductions in the capital gains tax rate for years. The capital gains tax rate is the tax rate applied to increases in stock prices (or capital gains). The argument put forward is that if people retain a higher percentage of their capital gains then private saving ($S_{pvt}$) and the supply of capital would increase. The following asks you to use the long-run, static model of section II to trace through the effects of a capital gains tax cut. Assume that the economy is closed so that $NX = 0$. **Note: parts a and b required the use of diagrams.**

a. In the capital market, the capital gains tax cut shifts the supply of capital curve to the right. This lowers the real rental price of capital ($R/P$) and increases the quantity of capital ($K$). In the production function with capital on the x-axis, the economy moves along the curve which increases the level of real GDP ($Y$) in the long-run.

In the labor market, the capital gains tax cut increases the marginal product of labor. This shifts the labor demand curve to the right which raises the real wage rate ($W/P$) and leaves the quantity of labor ($L$) the same. In the production function with labor on the x-axis, the increase in the marginal product of labor shifts the production function up and thus the same quantity of labor produces more real GDP ($Y$) in the long-run.

b. The capital gains tax cut will increase private saving ($S_{pvt}$), but decrease public saving ($S_{govt}$). The net effect is most likely that national saving ($S = S_{pvt} + S_{govt}$) will decrease. In the closed-economy loanable funds market, the decrease in national saving shifts the national saving curve to the left. This causes the real interest rate ($r$) to rise which in turn decreases real investment ($I$) in the long-run.

c. The quantity theory of money states that the price level ($P$) in the long-run is determined by the following equation: $P = \frac{M}{Y} \cdot V$. Since velocity ($V$) is constant and the Federal Reserve has done nothing to the supply of money ($M$), the price level will fall due to the increase in the level of real GDP ($Y$) in the long-run.

d. Alan Greenspan, chairman of the Federal Reserve, is unable to increase the long-run level of real GDP ($Y$) by printing money. The reason is because money is neutral in the long-run. In other words, a change in a nominal factor like the quantity of money has no effect on real variables like real GDP.

4. Suppose that the U.S. economy is a small, open-economy. The following asks you to use the long-run, open-economy model of chapter 8 to trace through the effects of a capital gains tax cut. **Note: part a requires the use of a diagram.**
a. The net effect of a capital gains tax cut is most likely that national saving 
\( S = S_{pvt} + S_{govt} \) will decrease. In the small, open-economy loanable funds market, the 
decrease in national saving shifts the national saving curve to the left. Since the economy 
takes the world interest rate \( (r^*) \) as given, there is no change on the real interest rate \( (r) \) and 
real investment \( (I) \). However, with less national saving and the same amount of real 
investment, real net exports \( (NX) \) would decrease in the long-run. The decrease in real net 
exports shows up as an increase in foreign saving \( (S_{foreign}) \).

b. In the long-run, the capital gains tax cut will decrease national saving which will cause the real 
exchange rate \( (e) \) to increase. The increase in the real exchange rate decreases real net 
exports \( (NX) \). The nominal exchange rate \( (E) \) is equal to \( \frac{\epsilon \cdot P^*}{P} \). With the real exchange 
rate \( (\epsilon) \) higher and the domestic price level \( (P) \) lower, the nominal exchange rate must also 
rise.

5. Researchers have shown that a Solow growth model with a Cobb-Douglas production function 
of \( Y = K^{0.3} L^{0.7} \) can describe cross-country growth differences.

a. \( Y = \frac{K^{0.3} L^{0.7}}{L} = \frac{K^{0.3} L^{0.7}}{L^{0.7}} = \left( \frac{K}{L} \right)^{0.3} = k^{0.3} \)

b. In steady state, \( \Delta k = sf(k) - (\delta + n)k = 0 \). This implies that \( sk^{0.3} = (\delta + n)k \). Solving 
for \( k \), the steady-state capital stock per worker \( k^* = \left[ \frac{S}{(\delta + n)} \right]^{1/0.7} \). Substituting the value 
of \( k^* \) into \( f(k) \) gives us the steady-state output per worker 
\( y^* = (k^*)^{0.3} = \left[ \frac{S}{(\delta + n)} \right]^{0.3/0.7} \). Using the national saving identity, steady-state consumption 
per worker is 
\( c^* = (1-s)y^* = (1-s) \left[ \frac{S}{(\delta + n)} \right]^{0.3/0.7} \).
c. The country with a high initial capital stock per worker \( k_{0}^{\text{high}} \) (rich country) will grow slower than the country with a low initial capital stock per worker \( k_{0}^{\text{low}} \) (poor country). The poor country will have a higher marginal product of capital than the rich country. As a result, the poor country will have greater capital accumulation \( (\Delta k) \) and thus faster transitional growth.

d. The Golden Rule implies that the \( MPK = n + d \). Substituting in \( 0.3k^{-0.7} \) for \( MPK \) and 0.07 for \( n + d \), the condition becomes \( 0.3k^{-0.7} = 0.07 \). Solving this expression for \( k \) yields the Golden Rule level of capital per worker \( k_{\text{gold}}^* = (0.07 / 0.3)^{1/0.7} = (0.3 / 0.07)^{1/0.7} \approx 8 \)

e. The saving rate that would generate the Golden Rule level of capital per worker is found by \( sk^{0.3} = (\delta + n)k \). Substituting in 0.07 for \( n + d \) and 8 for \( k \), the condition becomes \( s8^{0.3} = (0.07)8 \). Solving this expression for \( s \) yields the Golden Rule saving rate as \( s_{\text{gold}} = (0.07)^{1/0.3} \approx .3 \) or 30 percent. Since policymakers can only influence the after-tax return on saving, it may be difficult to encourage people to save at that rate.