Risk, Finance, and the Volatility in Per Capita Growth Rates

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Abstract

This paper shows that the volatility in per capita growth rates may be explained by agents’ responses to uncertainty under different financial structures. Under a poorly-developed financial sector, agents must cover their consumption risk by investing directly in two assets: a one-period consumer loan and a two-period capital good. Due to incomplete markets and incomplete participation in those markets, the asset rates of return are not arbitraged together. As a result, the economy experiences wide fluctuations in the levels of net investment and output. Under a well-developed financial sector, agents can cover their consumption risk by depositing their savings in a bank. Competition in the banking sector forces each bank to make the same portfolio decision and thus post the same set of deposit rates in every time period. Therefore, the mean levels of net investment and output are higher, while the volatility is lower under a well-developed financial system. This result conforms with the empirical evidence that shows a negative connection between the initial level of financial development and the volatility in per capita growth rates.

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1. INTRODUCTION

During the past few years, growth economists have begun to focus on the volatility of per capita real GDP growth rates. Easterly et. al. (1993) argue that per capita growth rates are highly unstable over time, while country characteristics such as education, government consumption and finance measures are highly persistent. Furthermore, Acemoglu and Zilibotti (1997) find a negative link between the volatility of per capita growth rates and the initial level of per capita GDP. Lastly, this paper will show that there is a negative correlation between the volatility in per capita growth and the initial level of financial intermediation. These empirical results suggest the following three stylized “facts”: (1) volatility of growth rates cannot be fully explained by the volatility of country characteristics, (2) poorer economies experience greater volatility in per capita growth and (3) economies with less-developed financial sectors experience greater volatility in per capita growth.

This paper argues that these three “facts” of economic growth may arise from agents’ responses to uncertainty under different financial structures. Broadly speaking, agents under a poorly-developed financial sector cover risk mostly through direct holdings of assets, while agents under a well-developed financial sector use institutions to help pool their risk. As a result, rates of return on assets and thus investment rates tend to fluctuate more in an economy with poorly-developed financial sector. In a growth model, these fluctuations in investment will lead to increased volatility in per capita growth rates.

A model with the following six features is constructed. First, agents in the model are finite-lived. A four-period overlapping generations model is adopted to capture this feature. Second, agents desire to “smooth” their consumption path. As a result, agents in the model follow a typical life-cycle: borrow early in their lives, save during the middle, and dissave towards the end. Third, agents are subject to a liquidity risk during the end of their lifetime. This liquidity risk is modeled as an idiosyncratic shock which alters the preferences of the agent during the last two periods of their lifetime. Fourth, there are long delays between capital
expenditures and receipts. Cameron (1967) refers to this feature as “the slow cycle of production.” Fifth, there are no markets for capital in progress. This feature seems justified by the lack of equity markets in developing economies. Lastly, there are positive externalities in capital accumulation which allows for self-sustained growth in output. Arrow (1962) and Romer (1986) (1987) discuss the empirical plausibility of this feature.

The model is then solved under two financial structures: a poorly-developed financial sector where households must invest autarkically and a well-developed financial sector where households can invest through a deposit-issuing bank. Under the first structure, agents must cover their liquidity risk by investing in both a one-period consumer loan and a two-period capital good. The consumer loan rate rises with the growth in output since consumer lending takes place across generations, while the rental rate on capital remains stable due to positive externalities in production. As a result, the economy experiences wide fluctuations in the levels of investment and output. Under the financial structure, agents cover their liquidity risk by depositing their savings in a bank. Competition in the banking sector forces each bank to make the same portfolio decision and thus post the same set of deposit rates in every time period. As a result, the mean levels of investment and output are higher, while the volatility in investment and output is essentially eliminated. Therefore, the model predicts that those economies with a highly-developed financial sector should exhibit higher mean growth rates with less volatility than those economies with a poorly-developed financial sector.

This paper is related to the literature on credit and growth by Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), Levine (1991), Greenwood and Smith (1997), and Acemoglu and Zilibotti (1997).\footnote{For a review of this literature, see Pagano (1993) and Levine (1997).} As in these papers, we find a positive connection between financial intermediation and growth in per capita GDP. Like Bencivenga and Smith (1991) and Levine (1991), we show that this connection may result from the reduction in liquidity risk faced by households. However, since we endogenize the return on the one-period asset, we find
that this reduction in risk can also reduce the volatility in per capita growth rates. Greenwood and Jovanovic (1990) and Acemoglu and Zilibotti (1997) also find that financial intermediation reduces the volatility of growth rates through diversification of production risk. Our work differs from these authors in that we focus on consumption risk that possesses no aggregate uncertainty and we endogenize the rates of return on all assets.

The paper proceeds as follows. Section 2 presents empirical evidence on the connection between the initial level of finance and the volatility of growth. Section 3 constructs the model. Section 4 solves for the steady-state equilibria under financial autarky. Section 5 introduces the banks and then solves for the “intermediated” steady-state. Lastly, section 6 concludes.

2. THREE STYLIZED “FACTS”

The first stylized “fact” is that volatility of growth rates cannot be fully explained by the volatility of country characteristics. For the period 1960-1988, Easterly, et. al. (1993) find that per capita growth rates have a cross-decade correlation of 0.1 to 0.3, while the country characteristics have a cross-decade correlation of 0.6 to 0.9. Moreover, their cross-decade correlation for their measure of financial intermediation -- ratio of M2 to GDP -- is 0.75 and 0.95. Figure 1 plots the cross-decade correlation for four indicators of financial development suggested by King and Levine (1993). There is high persistence for all indicators with correlations ranging from 0.6 to 0.9. For this same sample of 69 countries, there is a cross-decade correlation of 0.4 for per capita growth rates. This evidence suggests that the correlation between mean levels of finance and growth found by King and Levine (1993) does not extend to the second moment.

The second stylized “fact” is that poorer economies experience greater volatility in per capita growth. For the period 1960-1985, Acemoglu and Zilibotti (1997) regress the standard deviation of per capita GDP growth rates on per capita GDP in 1960 and find a negative and highly-significant relationship. Does this negative relationship extend to the initial level of
financial intermediation? As figure 2 shows, the answer is yes. For a sample of 66 countries from 1960 to 1990, we find a negative and significant link ($t$-statistic of $-4.08$ and R-square of 0.21) between the standard deviation of per capita GDP growth rates and the ratio of liquid liabilities to GDP in 1960. For this same sample, we estimate the regression of Acemoglu and Zilibotti (1997) and get similar results ($t$-statistic of $-4.36$ and R-square of 0.23). Furthermore, when the regression is run with both initial conditions, each variable remains negative and significant at the 1% level. This evidence suggests a third stylized “fact” that economies with poorly-developed financial sectors experience higher volatility in per capita growth rates.

3. THE MODEL

a. The Physical Environment

The model is populated by a sequence of four-period-lived, overlapping generations. Members of each generation have identical preferences and differ only by date of birth. We assume there is no population growth and normalize the population of each cohort to one.

The age or period of life of an agent is indexed by $i$ ($i=0,1,2,3$), while his generation or date of birth is denoted by $τ$. For example, $c_{1,τ}$ would denote age-1 consumption by the generation born at $τ$. Therefore, the current time in the model is equal to the sum of $i$ and $τ$.

There are two goods in the economy: a single consumption good and a single capital good. The consumption good is a type of composite commodity and is produced from capital and labor. For reasons to be discussed shortly, production is undertaken by a subset of age-3 agents. Moreover, the consumption good can be traded or stored for one period at rate $n$.

Through the application of a two-period technology, the consumption good can also be converted into the capital good. Specifically, this capital-producing technology converts one unit of the consumption good into $R$ units of the capital good where $R$ is exogenous and net of all costs of production. The capital good is assumed to be a completely divisible “putty” and depreciate totally after one period. If this process is interrupted after one period, then none of
the initial investment is salvageable.\(^2\) Furthermore, both the capital itself and the rights of future capital cannot be traded.

For example, one may think of the consumption good as kernels of corn. The corn can either be stored in the silo or planted in the ground. If stored each kernel of corn returns \(n\) kernels after one period. On the other hand, if the kernel is planted, it produces \(R\) stalks of corn in two periods. These stalks of corn are the capital goods since labor is required to harvest the corn. However, after one period, the planted kernels are neither consumable nor tradable.

b. Endowments and Preferences

At the beginning of age 1, each agent is endowed with one unit of labor. This is inelastically supplied in exchange for a competitive wage \(w_{1,t}\), paid in units of the consumption good. An agent receives no other endowments during his lifetime.

All agents maximize their lifetime utility from the vantage point of age 0. Letting \(c_{i,t}\) denote age-\(i\) consumption by generation \(t\), all agents have the utility function

\[
u(c_{0,t}, c_{1,t}, c_{2,t}, c_{3,t}; \phi) = \ln c_{0,t} + \ln c_{1,t} + \ln(c_{2,t} + \phi c_{3,t})
\]

(1)

where \(\phi\) is an individual-specific random variable that is realized at the beginning of age 2. \(\phi\) has a probability distribution of

\[
\begin{align*}
0 & \quad \text{with probability } 1 - \pi \\
1 & \quad \text{with probability } \pi.
\end{align*}
\]

(2)

An individual agent’s \(\phi\) is assumed to be private information, while the probability distribution of \(\phi\) is publicly known. Moreover, there exists no credible way for an agent to

\(^2\) A more general specification would have the capital-producing technology return \(x\) units of the consumption good after one period, where \(0 \leq x < n\). By assuming that the capital-producing process is completely irreversible \((x = 0)\), we still retain the idea that there are readily discernible costs to early liquidation and substantially simplify the solution in section III. Bhattacharya and Gale (1987), Jacklin and Bhattacharya (1988) and Bencivenga and Smith (1992) make the same irreversibility assumption.
transmit his draw of $\phi$ to others. As a result, traditional Arrow-Debreu insurance contracts are not available in this economy.

Since all agents know $\pi$, there is no aggregate uncertainty or risk in the model. In other words, each agent knows that $1-\pi$ members of every generation will have a draw of 0 from the random variable $\phi$, while $\pi$ agents will have a draw of 1. As a result, every agent can perfectly predict his future wage rate and all future rates of return. What the agent lacks, however, until age 2 is knowledge of his realization of $\phi$.

The following preference structure captures the Diamond-Dybvig notion of “desire for liquidity.” At age 2, the agent acts in one of two ways. If $\phi = 0$, the agent suddenly and more importantly, unexpectedly, experiences a need for liquidity. In response to this “shock” he wants to consume all of his wealth at age 2. These agents are referred to as “early consumers.” However, if $\phi = 1$, the agent experiences no liquidity shock and thus is indifferent between consuming his wealth at the age of 2 or 3. These are the “late consumers.”

By adding the initial time period to the agent’s lifetime, we introduce an explicit borrowing decision to the Diamond-Dybvig framework. Since agents receive no endowment at age 0, they must borrow resources for first-period consumption. Moreover, since agents receive no endowment at ages 2 and 3, they must save for later consumption. Because the agent does not know his type, this savings will be held in both one- and two-period assets. Therefore, we create a demand for and a supply of a one-period asset that transfers resources across generations.

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3 In their original work, Diamond and Dybvig (1983) model the late consumers valuing only last-period consumption. Under this specification, equation (1) would be: $\ln c_{0,0} + \ln c_{1,0} + [(1-\phi) c_{2,0} + \phi c_{3,0}]$. Other authors, most notably Jacklin (1987) and Jacklin and Bhattacharya (1988), model both types of agents valuing consumption at age 2 and 3, but varying in their time preferences: early consumers preferring more consumption at age 2 and late consumers preferring more consumption at age 3. Under this specification, equation (1) would be: $\ln c_{0,0} + \ln c_{1,0} + (1-\phi)(\ln(c_{2,0} + \gamma_0 c_{3,0}) + \phi \ln(c_{2,0} + \gamma_1 c_{3,0}))$ where $\gamma_1 > \gamma_0$. Our specification is used by Bencivenga and Smith (1991) (1992) and Levine (1991). All three specifications deliver similar asset demand functions, but only (1) yields closed-form solutions in section 4.
c. Assets Available

The following table lists the three assets available at any time $t$. The first asset is the storage technology. For each unit of the consumption good, storage returns $n$ units of the consumption good after one period. The second asset is a one-period loan to the age-0 agents. For each unit of the consumption good, consumer lending returns $r_t$ units of the consumption good after one period. The subscript $t$ refers to the time period when the loan is negotiated. We assume that agents always repay their loan in full and can only lend once per lifetime.\(^\text{4}\)

<table>
<thead>
<tr>
<th>Time Period</th>
<th>$t$</th>
<th>$t + 1$</th>
<th>$t + 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storage</td>
<td>-1</td>
<td>$n$</td>
<td>0</td>
</tr>
<tr>
<td>Consumer Lending</td>
<td>-1</td>
<td>$r_t$</td>
<td>0</td>
</tr>
<tr>
<td>Capital Investment</td>
<td>-1</td>
<td>0</td>
<td>$R\rho_{t+2}$</td>
</tr>
</tbody>
</table>

The last asset is investment into the capital-producing technology. When one unit of the consumption good is invested at time $t$, the two-period technology returns $R\rho_{t+2}$ units of the consumption good at $t + 2$. As mentioned earlier, $R$ is the two-period conversion rate of a unit of the consumption good into capital goods. The term $\rho_{t+2}$ is the return or rent on one unit of capital from the production of consumption goods at time $t + 2$. However, if the investment project is prematurely liquidated at $t + 1$, it returns no consumption goods.

\(^{4}\) Since all wage rates are known, the first assumption basically ensures that there are no enforcement problems in the consumer loan market. The second assumption prevents agents from re-lending the return on either storage or consumer lending and thus forces the late agents to consume these returns at age 2. Even though the term structure of assets may seem extreme, the model does capture the notion that agents face real costs by liquidating capital early and reduced consumption by investing in storage or lending.
d. The Consumer Loan Market

Let \( r^* \) represent the gross interest rate that equates the supply of and demand for consumer loans and \( r_t' \) represent the gross interest rate that actually prevails in the market. Moreover, let \( g' \) and \( h' \) represent the percentage of labor income \( w_{1,t} \) an agent born at \( t \) devotes to consumer lending and consumer borrowing, respectively. At each time \( t \), two exchanges occur. First, the age-1 agents (generation \( t - 1 \)) lend consumption goods to the age-0 agents (generation \( t \)) at \( r_t' \). Second, the age-1 agents settle their debt with the age-2 agents (generation \( t - 2 \)). These two exchanges are represented as

\[
\begin{align*}
  w_{1,t-1} g' & = w_{1,t-1} g'^{-1} \quad \text{(3)} \\
  w_{1,t-1} h'^{-1} & = w_{1,t-2} g'^{-2} \quad \text{(4)}
\end{align*}
\]

Equation (3) determines the market-clearing, consumer loan rate \( r^* \) at time \( t \). Equation (4) determines consumption for the age-1 and age-2 generations at time \( t \).

e. Production and the Labor Market

The supply side of the economy follows Bencivenga and Smith (1991). The \( \pi \) agents of generation \( t \) who do not realize a liquidity shock become entrepreneurs at the age of 3. These entrepreneurs use only their inherited stock of capital \( k_{t+3} \) for production. Since all agents are identical ex ante at age 1, all entrepreneurs have the same stock of capital. Let \( l_{1,t+2} \) denote the age-1 workers of the younger generation \( t + 2 \) employed by each entrepreneur. Entrepreneurs produce consumption goods according to the following production function

\[
y_{t+3} = \bar{k}_{t+3}^\delta k_{t+3}^\theta l_{1,t+2}^{1-\theta} \quad \text{where} \quad \theta \in (0,1) \quad \text{and} \quad \delta = 1 - \theta.
\]

The \( \bar{k}_{t+3} \) is the “average per entrepreneur capital stock” at time \( t + 3 \). As in Arrow (1962) and Romer (1986), \( \bar{k}_t \) is meant to capture the positive externalities or spillovers in knowledge.
or technology. Consequently, an individual firm knows of the externality, but does not take it into account when choosing his level of inputs. However, if any one entrepreneur increases his own level of capital, then all entrepreneurs benefit with an increase in production.

The labor market is competitive. Since all age-1 individuals of generation $t + 2$ supply labor and the population of each generation equals one, the total labor supply equals one. However, only $\pi$ of generation $t$ become entrepreneurs and thus demand labor. Therefore, equilibrium in the labor market implies that

$$\pi l_{t,t+2}^d = 1. \tag{6}$$

Setting $l_{t,t+2}^d$ equal to the marginal product of labor and using the equilibrium condition $k_{t+3} = \bar{k}_{t+3}$, the market-clearing wage at time $t + 3$ is

$$w_{1,t+2} = \bar{k}_{t+3}(1 - \theta)\pi^0. \tag{7}$$

With $\pi$ and $\theta$ exogenous, growth in the real wage equals growth in the stock of capital.

Substituting equations (5) - (7) into the per firm profit function, $y_{t+3} - l_{1,t+2}w_{1,t+2}$, and then dividing by $k_{t+3}$, the return on a unit of capital at $t + 3$ is

$$\rho_{t+3} = \rho = \theta \pi^{\delta - 1}. \tag{8}$$

See appendix A for the derivation. Since $\delta = 1 - \theta$, there are constant returns to capital at the social level. This constancy of returns allows for the possibility of self-sustained growth.

Furthermore, we assume that both the storage rate and consumer loan rate at time 0 can never exceed the expected return on capital $\pi R \rho$:

$$n < R \theta \pi^0 \tag{9}$$

$$\frac{\bar{k}_1}{\bar{k}_0} < R \theta \pi^0 \tag{10}$$
where \( g^{-1} = 1/3 \) and \( h^0 = 1/3 r_0 \) were used in (10). These two conditions place restrictions on the three parameters \( \{R, \theta, n\} \) and two initial conditions \( \{\bar{k}_0, \bar{k}_1\} \) and ensure that capital is held at time 0.

Returning to the corn example, the \( \pi \) agents who do not realize a liquidity shock become the entrepreneurial farmers at age 3. Each farmer arrives at age 3 with \( k_{t+3} \) stalks of corn. With this capital, the farmer hires \( l_{t+3} \) age-1 workers to harvest the corn. Production takes place and each worker receives \( w_{t+2} \) kernels of corn as wage payment, while the farmer receives \( k_{t+3} \rho \) kernels of corn as payment for his stock of capital.

### f. General Equilibrium

There are four conditions that must be met for a (static) general equilibrium at any time \( t \).

1. The labor market must clear according to (6).
2. The consumer loan market must clear according to (3) and (4).
3. The total stock of capital at \( t \) must identically equal those resources invested into the capital-producing technology at time \( t - 2 \) that remained uninterrupted

\[
\pi \bar{k}_t \equiv \alpha_t w_{t+3} e^{\rho t} R \tag{11}
\]

where \( e^{\rho t} \) represents the percentage of income devoted to capital by generation \( t - 3 \) and \( \alpha_t \) denotes the percentage of investment that remained uninterrupted until \( t \).

4. In the commodity market, the total supply of consumption goods at time \( t \) must equal consumption by all four overlapping generations plus total investment by generation \( t - 1 \). See appendix A for the derivation.

The first two conditions determine \( w_{t+1}, r^*_t, c_{0,t} \) and \( c_{1,t-1} \). The third condition determines the two-period growth rate of capital \( \mu_t \), while the fourth is dropped using Walras’ Law.
4. FINANCIAL AUTARKY

a. Consumer Optimization

Let \( \{ e^t, f^t, g^t, h^t \} \) represent the percentage of labor income \( w^t_{1,t} \) an agent born at \( t \) devotes to capital, storage, consumer lending and consumer borrowing, respectively. Recall that the agent receives no endowment during age 0. As a result, first-period or age-0 consumption is limited to the level of consumer borrowing \( w^t_{1,t}h^t \). At age 1, the agent supplies his unit of labor to the entrepreneurs and receives \( w^t_{1,t} \). With this income, the agent settles his consumer debt \( w^t_{1,t}h^t r^t \); allocates his portfolio between capital holdings \( w^t_{1,t}e^t \), storage \( w^t_{1,t}f^t \), and consumer lending \( w^t_{1,t}g^t \); and consumes the remainder \( w^t_{1,t}(1-e^t-f^t-g^t-h^t r^t) \).

At the beginning of age 2, the agent realizes whether he is an early consumer (\( \phi = 0 \)) or a late consumer (\( \phi = 1 \)). At this time, the agent consumes the stored goods \( w^t_{1,t}f^t n \) and the return on the consumer loan \( w^t_{1,t}g^t r^t \) regardless of his type. If the individual is an early consumer, he liquidates his holdings of capital and receives nothing. He does so because he can neither consume the immature capital nor sell the rights to it. If the agent is a late consumer, however, he maintains his capital investments until age 3. At age 3, the late agents become the \( \pi \) entrepreneurs. Each entrepreneur hires age-1 labor, produces the consumption good, pays labor, and consumes the two-period return on his stock of capital \( w^t_{1,t}e^t R \rho_{t+3} \).

Recall that since \( \pi \) is known each agent can perfectly forecast all future wage rates and consumer loan rates. Taking \( w^t_{1,t}, r^t_i, r^t_{i+1} \) and \( \rho \) as given (and known), the agent of generation \( t \) chooses his decision variables \( \{ e^t, f^t, g^t, h^t \} \) to maximize his ex ante expected lifetime utility

\[
E(u) = \ln \{ w^t_{1,t}(h^t) \} + \ln \{ w^t_{1,t}(1-e^t-f^t-g^t-h^t r^t) \} + (1-\pi) \ln \{ w^t_{1,t}(f^t n + g^t r^t_{i+1}) \} + \pi \ln \{ w^t_{1,t}(e^t R \rho + f^t n + g^t r^t_{i+1}) \} 
\]

subject to
\[ e', f', g', h' \geq 0 \quad (13) \]
\[ (e' + f' + g' + h' r'_t) \leq 1. \quad (14) \]

The first-order conditions with respect to \( e', f', g', h' \) are

\[
\frac{1}{1 - e' - f' - g' - h' r'_t} = \frac{\pi R \rho}{e' R \rho + f' n + g' r'_{t+1}} \quad (15)
\]

\[
\frac{1}{1 - e' - f' - g' - h' r'_t} = \frac{(1 - \pi)n}{f' n + g' r'_{t+1}} + \frac{\pi n}{e' R \rho + f' n + g' r'_{t+1}} \quad (16)
\]

\[
\frac{1}{1 - e' - f' - g' - h' r'_t} = \frac{(1 - \pi)r'_{t+1}}{f' n + g' r'_{t+1}} + \frac{\pi r'_{t+1}}{e' R \rho + f' n + g' r'_{t+1}} \quad (17)
\]

\[
\frac{r'_t}{1 - e' - f' - g' - h' r'_t} = \frac{1}{h'} \quad (18)
\]

where (13) and (14) must hold. When deciding upon his holdings of assets in (15) - (17), the agent equates the marginal utility lost by foregoing age-1 consumption with the expected marginal utility gained in later consumption. Similarly, in condition (18), consumer borrowing takes place until the marginal utility lost in age-1 consumption equals the marginal utility gained in age-0 consumption. In conditions (16) and (17), the consumer loan rate \( r'_{t+1} \) need not be arbitraged to the storage rate \( n \) since the storage technology cannot be issued \( (f' \geq 0) \).

Since the agent does not know his type and therefore, his consumption sequence, when allocating his portfolio, he selects both one- and two-period assets. Consumer lending and storage offer the agent a higher return if he turns out to be an early consumer, while capital offers a higher return if he turns out to be a late consumer. However, since both consumer lending and storage offer a riskless return, the agent will hold the asset that has the greater return. What separates this work from Bencivenga and Smith (1991), Levine (1991) and others is that the return on the one-period asset is not exogenously set, but rather is determined by the growth rate of the economy.
b. The Consumer Loan Market and Static Equilibria

In figure 3, there is a diagram of the consumer loan market at time $t + 1$. The “prevailing” loan rate $r^{l}_{t+1}$ is placed on the vertical axis and the quantity of loans $l$ is mapped on the horizontal axis. Recall that $r^{*}_{t+1}$ represents the loan rate that equates the supply and demand for loans over the region $(0, \infty)$. The demand for loans $l^{d}$ is a downward-sloping rectangular hyperbola. Since both storage and consumer lending offer a riskless, one-period return, the supply of loans $l^{s}$ is an upward sloping line that begins at $n$.

There are three possible equilibria determined by the value of $r^{*}_{t+1}$. First, if $r^{*}_{t+1}$ is below $n$, as in $l^{d}(1)$, then $r^{l}_{t+1}$ equals $n$. In this equilibrium, the borrowing generation would bid $r^{l}_{t+1}$ up to $n$ in order to secure consumer loans. As a result, the quantity of loans $l$ would equal $l^{\min}$ with the excess supply of loans $(l^{\min} - l^{\min})$ being placed in storage. We refer to this equilibrium as the corner solution.\footnote{Bernanke and Gertler (1989) obtain a similar result for a business credit market. However, since the lenders and borrowers come from the same generation in their model, their result is obtained simply by assuming that the proportion of lenders from each generation is large.} Second, if $r^{*}_{t+1}$ falls between $n$ and $\pi R \rho$, as in $l^{d}(2)$, then $r^{l}_{t+1}$ equals $r^{*}_{t+1}$. Since storage must be non-negative, the borrowing agents cannot arbitrage $(r^{*}_{t+1} - n)$ away by issuing storage to purchase consumer loans. Similarly, since capital pays out only after two periods, the age-1 agents cannot arbitrage $(\pi R \rho - r^{*}_{t+1})$ away by borrowing resources to purchase capital. We refer to this equilibrium as the interior solution.

Lastly, if $r^{*}_{t+1}$ exceeds the expected return on capital $\pi R \rho$, as in $l^{d}(3)$, then $r^{l}_{t+1}$ also equals $r^{*}_{t+1}$ and $l$ would equal $l^{\max}$. With $r^{l}_{t+1} > \pi R \rho$, no investment in capital occurs in this equilibrium. However, there is an incentive for the age-2 and age-3 agents to sell their holdings of capital in order to issue loans. Two frictions prevent them from doing so. First, since there are no ex post markets for capital in progress, the age-2 agents cannot sell shares of their future capital. Second, since agents only live for four periods, the age-3 entrepreneurs will not sell their
current capital for an asset that pays out next period. Because of these incomplete markets and participation in those markets, the prevailing loan rate $r_{t+1}$ is not bounded from above. We refer to this equilibria as the stagnation solution.

c. Steady-State Equilibrium

Having described the three static equilibria, we now examine the two steady-state growth paths. To simplify the discussion, we adopt the representation proposed by Blanchard and Kahn (1980):

$$\begin{bmatrix} \bar{k}_{t+2} \\ \bar{k}_{t+1} \end{bmatrix} = \begin{bmatrix} A & B \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \bar{k}_{t+1} \\ \bar{k}_t \end{bmatrix}$$

(19)

where $A$ and $B$ are functions of the three parameters \{R, \theta, n\}. The functions $A$ and $B$ are determined by the general equilibrium conditions (3) - (11) and the first-order conditions (15) - (18). Let $\lambda_1$ and $\lambda_2$ represent the two eigenvalues of (19) and $\mu_t$ represent the two-period growth rate in capital $\left(\bar{k}_{t+2} / \bar{k}_t\right) - 1$.

**Definition 1:** An autarkic steady-state growth path is a sequence of portfolio decisions \{ $e^t = e^{t+1} = e^{t+2} = \ldots$ \}, \{ $f^t = f^{t+1} = f^{t+2} = \ldots$ \}, \{ $g^t = g^{t+1} = g^{t+2} = \ldots$ \}, and \{ $h^{t-1} = h^t = h^{t+1} = \ldots$ \} on the part of agents that yields

$$\lim_{t \to \infty} \mu_t = \mu_{ss}$$

where $\mu_{ss} \in \mathbb{R}$.

The following two propositions describe the steady-state growth paths.

**Proposition 1:**

If $$\left( \frac{R\theta\pi^{\theta-1} - n}{n(1 - \pi)R\theta\pi^{\theta-1}} \right) \leq \frac{\bar{k}_0}{\bar{k}_1} \leq \left( \frac{n}{R\theta\pi^{\theta-1} - n} \right) \left( \frac{3(1 - \pi)(R\theta\pi^{\theta-1} - n)}{\pi(1 - \theta)(R\theta\pi^{\theta} - n)} \right),$$

then

$$A = 0, \quad B = \frac{(1 - \theta)\pi^\theta (\theta\pi^\theta R - n)R}{3(\theta\pi^{\theta-1}R - n)}$$
\[ \lambda_1 = \sqrt{B} , \quad \lambda_2 = -\sqrt{B} \]

\[ \mu_{ss} = B - 1 \]

The first inequality implies that \( r_0^* \leq n \), while the second inequality implies that \( r_1^* \leq n \).

Appendix B proves that these two inequalities are both necessary and sufficient for the steady-state. Under proposition 1, the economy begins and remains at the corner solution. Since \( B \) is a function of the parameters, there can be either negative steady-state growth \( (B < 1) \) or positive steady-state growth \( (B > 1) \).

**Proposition 2:**

If \( \left( \frac{k_0}{k_1} \right) < \left( \frac{R\theta\pi^{\theta-1} - n}{n(1-\pi)R\theta\pi^{\theta-1}} \right) \) and \( \left( \frac{n(1-\pi)R\theta\pi^\theta}{R\theta\pi^\theta - \pi n} \right) < (\lambda_1)^2 < R\theta\pi^\theta \), then

\[ A = -\frac{(1-\theta)\pi}{3\theta}, \quad B = \frac{(1-\theta)\pi^{\theta+1}R}{3} \]

\[ \lambda_1 = \frac{(1-\theta)\pi}{6\theta}\left[\sqrt{1 + \frac{12R\theta^2\pi^{\theta-1}}{(1-\theta)} - 1} \right] , \quad \lambda_2 = -\frac{(1-\theta)\pi}{6\theta}\left[\sqrt{1 + \frac{12R\theta^2\pi^{\theta-1}}{(1-\theta)} + 1} \right] \]

\[ \mu_{ss} = (\lambda_1)^2 - 1 \]

Under proposition 2, the steady-state equilibrium is a saddle-point where \( \bar{k}_1 = \lambda_1\bar{k}_0 \) is the stable arm. Condition \( (i) \) implies that \( r_0^* > n \) and therefore the economy begins in the interior solution. Condition \( (ii) \) ensures that growth along the stable arm \( \mu_{ss} \) is high enough to keep \( r_1^* > n \) and low enough to keep \( r_1^* < \pi R \theta \). If condition \( (ii) \) did not hold, then the economy would be immediately placed at the stagnation solution. See appendix B for details. As before, there can be either negative or positive steady-state growth depending upon the value of \( (\lambda_1)^2 \). However, since \( r_1^* > n \) under proposition 2, the steady-state growth rate under proposition 2 is less than the growth rate under proposition 1.

Even though the conditions for both propositions are nonlinear functions of \( \{R, \theta, n\} \) and \( \{\bar{k}_0, \bar{k}_1\} \), we can still discuss the probability of meeting either proposition 1 or 2. First, if \( \bar{k}_1 \)
increases relative to $\bar{k}_0$, then the probability of proposition 2 rises, while probability of proposition 1 falls. However, if $\bar{k}_0$ increases relative to $\bar{k}_1$, then the probability of both propositions fall. The effect of the three parameter values are best looked at as the difference between the expected return on capital to the storage rate: $\theta \pi^0 R - n$. If $n$ increases relative to $\theta \pi^0 R$, the probability of proposition 1 rises, while the probability of proposition 2 falls. However, if $\theta \pi^0 R$ increases relative to $n$, then proposition 2 becomes more likely. Unless $\pi$ is extremely low, it seems more likely that the parameters are such that proposition 2 holds.

d. Stability

Under proposition 1, the economy is stable. Since $A = 0$, there are two real roots of equal absolute value. As appendix B shows, the coefficient in front of the positive root exceeds the coefficient in front of the negative and thus the system will not oscillate. If $B < 1$, then the stock of capital and output converge to zero. If $B > 1$, then the two variables grow at the positive rate of $B - 1$ through time. As in Bencivenga and Smith (1991) and Levine (1991), this stability occurs because the return on the one-period asset is fixed at $n$, which makes investment depend only upon the current stock of capital. As a result, $\bar{k}_{t+1}$ is a function of $\bar{k}_t$ and not $\bar{k}_{t+1}$ ($A = 0$).

Under proposition 2, the economy is unstable except if it begins on the stable arm. Recall that there are two distinct real roots with the negative root $\lambda_2$ being the dominant root. Along the stable arm, the coefficient in front of unstable root $\lambda_2$ equals zero and the capital stock follows the path $\bar{k}_t = \bar{k}_0 (\lambda_1)'$. As in proposition 1, the capital stock and output either converge to zero or grow at the positive rate of $(\lambda_1)^2 - 1$. However, if the economy begins on any unstable branch, the path of capital will eventually begin to oscillate around zero. These oscillations place the economy at the stagnation solution where growth in both capital and
output ceases. These movements in capital along any unstable branch will register as marked fluctuations in per capita output.\textsuperscript{6}

This instability occurs because the one-period return on consumer lending is not bounded from above, while the two-period return on capital is bounded from below. Because consumer lending exerts a negative influence on capital formation \((A < 0)\), the unstable root \(\lambda_2\) is the dominant root under proposition 2. This unstable root causes the fluctuations in \(\bar{k}_t\).

More importantly, there is no mechanism to force the economy back to the stable arm. Recall that agents are unable to arbitrage the differences \((\pi R \rho - r_{t+1}^l)\) and \((r_{t+1}^l - \pi R \rho)\) away due to two-period capital formation and incomplete markets, respectively. Therefore, the consumer loan rate is free to fluctuate which drives the oscillations in investment and thus output.

Furthermore, the constant returns to scale in capital also eliminates another mechanism that could stabilize the path of capital. If there were decreasing returns \((\delta < 1 - \theta)\), then capital accumulation would decrease over time as the return on capital \(\rho\) fell. This would cause the wage rate of the borrowing generation to fall relative to the wage rate of the lending generation and thus push the consumer loan rate downward. However, since decreasing returns creates a nonlinearity in the capital accumulation equation, we cannot solve for closed-form solution and thus are unsure if there would be stability along the non-saddlepoint paths.\textsuperscript{7}

5. FINANCIAL INTERMEDIATION

a. Banking Technology

Assume there exists a technology that allows a coalition of agents to pool their wealth. By operating this “banking” technology, the coalition can allocate its resources between

\textsuperscript{6} If the investment function (11) contained an autonomous component \(a\), then the path of capital along an unstable branch would eventually oscillate around \(a\). If \(|\lambda_2| < 1\), then these oscillations dampen over time and the capital stock would converge to \(a\). If \(|\lambda_2| > 1\), then these oscillations increase over time and the economy would enter the stagnation solution.

\textsuperscript{7} Under decreasing returns, equation (19) becomes \(\bar{k}_t + A k_{t-1}^{\pi \rho} \bar{k}_t^{\pi \rho} + B k_{t-2}^{\pi \rho} = 0 \) where \(A < 0\) and \(B > 0\).
investment in capital, consumer lending and possibly storage. In exchange, each individual in the coalition receives a fixed claim on his deposit that is redeemable after one or two periods.

Moreover, assume the technology is freely available to all agents and is costless to use. These assumptions ensure there will be perfect competition in the banking industry. As a result, no individual coalition can exact rents or economic profits through the use of the technology. Furthermore, all activities of the coalition and its members are assumed to be public information.\(^8\)

With agents receiving their income at age 1, coalitions of age-1 agents form to operate the banking technology. These banking coalitions or “banks” offer their members gross rates of return of \(r^1_t\) and \(r^2_t\) for deposits received at time \(t\) and redeemed by agents after one and two periods, respectively. With these resources the banks invest in the capital-producing technology and lend to the age-0 agents at the rate \(r^l_t\). As before \(r^l_t\) is the “prevailing” or “equilibrium” gross interest rate for consumer loans issued at \(t\) and repaid at \(t + 1\).

Given the assumptions of the model, the number of coalitions that form is indeterminate. Henceforth, we talk of one coalition or “bank” for each generation. This bank receives the wealth of the age-1 agents and posts one set of deposit rates. However, in the consumer loan market, the bank is a price-taker due to the twin assumptions of no rigidities in the loan market and perfect competition in the banking industry.

b. Consumer Optimization

Let \(d^t\) represent the percentage of labor income \(w^1_t\), an agent born at \(t\) deposits in the bank. As under autarky, the agent borrows \(w^1_t, h^t\) at age 0. At age 1, the agent supplies his labor, receives \(w^1_t\), settles his consumer debt, deposits \(w^1_t, d^t\) and consumes the remainder.

\(^8\) This assumption accomplishes two objectives. First, it eliminates the need for coalition members to “monitor” the actions of the coalition itself. See Diamond (1984) for a model where the actions of the intermediary are private information. Second, it eliminates the possibility of a Diamond-Dybvig type bank-run where late agents withdraw after one period because they fear that other late agents are withdrawing at that time. See Freeman (1988) and Qi (1994) for a dynamic Diamond-Dybvig model where bank runs can occur.
If the agent is an early consumer, he withdraws at age 2 and receives \( w_{1,t}d'r_{t+1}^1 \). If the agent is a late consumer, he withdraws at age 3 and receives \( w_{1,t}d'r_{t+1}^2 \) if and only if \( r_{t+1}^1 \leq r_{t+1}^2 \).

Assuming that all of the coalition member’s wealth must be deposited in the bank, the age-0 agent of generation \( t \) selects \( \{d', h'\} \) to maximize his ex ante expected lifetime utility

\[
E(u) = \ln \{ w_{1,t}(h') \} + \ln \{ w_{1,t}(1-d' - h'r') \} + \\
(1-\pi) \ln \{ w_{1,t}(d'r_{t+1}^1) \} + \pi \ln \{ w_{1,t}(d'r_{t+1}^2) \}
\]  

subject to \( d', h' \geq 0 \). The solution to the problem is to set

\[
h' = \frac{1}{3r'} 
\]  

\[
d' = \frac{1}{3}.
\]  

By depositing his wealth in the bank, the agent allows the bank to determine his and the rest of the coalition’s portfolio in exchange for a higher expected lifetime utility.

c. Objective of the Bank

The objective of the banking coalition is to maximize the welfare of its members or depositors. Since all members of the coalition are identical ex ante, this objective reduces to maximizing the expected utility of the representative depositor:

\[
E(u) = (1-\pi) \ln \{ w_{1,t}(d'r_{t+1}^1) \} + \pi \ln \{ w_{1,t}(d'r_{t+1}^2) \}
\]  

The maximand is the expected utility at ages 2 and 3 since the age-1 agents form the coalition.

Let \( q^{t+1} \) and \( (1-q^{t+1}) \) denote the percentage of deposits invested in capital and supplied to the consumer loan market at \( t + 1 \). The banking coalition faces the following constraints:

\[
(1-\pi)r_{t+1}^1 \leq (1-q^{t+1})r_{t+1}^l
\]  

\[
\pi r_{t+1}^2 \leq q^{t+1} R \rho
\]
Budget constraints (24) and (25) stipulate that the \((1-\pi)\) early consumers are paid by the return on consumer lending and the \(\pi\) late consumers are paid by the return on capital. If the loan market is at the corner solution, then the bank lends and stores resources at the rate \(n\). Equation (26) induces the early consumers to withdraw after one period and the late consumers to withdraw after two periods. Equation (27) prevents the age-0 agents from depositing for arbitrage profits. These two self-selection constraints ensure that only the early consumers withdraw after one period. Lastly, to receive deposits, the bank must set the set the deposit rates to guarantee its members a greater expected lifetime utility than under financial autarky.

Substituting the budget constraints (24) and (25) into (23), the banking coalition of generation \(t\) selects \(q^{t+1}\) to maximize

\[
E(u) = (1-\pi) \ln \left\{ \frac{w_{t,t} (1-q^{t+1}) r_{t+1}}{3(1-\pi)} \right\} + \pi \ln \left\{ \frac{w_{t,t} q^{t+1} R \rho}{3\pi} \right\}
\]

subject to (26) and (27) and that welfare is higher than under financial autarky. Note that because there is perfect competition in the banking industry the two budget constraints bind.

The interior solution is \(q^{t+1} = \pi\). At this solution, the bank allocates \((1-\pi)\) and \(\pi\) of its resources to the agents who withdraw after one and two periods, respectively. Therefore, if agents self-select, the bank offers a set of deposit rates \(\{r_{t+1}^1, r_{t+1}^2\}\) equal to \(\{t_{t+1}^1, R \rho\}\). Since the ex ante return dominates the return under autarky, agents deposit all of their wealth in the bank.

d. Steady-State Equilibrium

Definition 2: an “intermediated” steady-state growth path is a sequence of portfolio decisions \(\{q^t = q^{t+1} = q^{t+2} = \ldots\}\) on the part of the bank and deposit decisions \(\{d^{t+1} = d^t = d^{t+1} = \ldots\}\) on the part of agents that yields \(\{r^t, r^1_t, r^2_t\}\) and \(\mu_t\) such that
(i) all savings is deposited  
(ii) all early agents withdraw after one period  
(iii) all late agents withdraw after two periods  
(iv) the ex ante utility of depositors is maximized.

The first condition ensures that the bank intermediates all the wealth of the economy. The second and third conditions ensure that only the \((1-\pi)\) and \(\pi\) members of the banking coalition withdraw after one and two periods, respectively. This allows the banks to take advantage of the aggregate certainty embodied in \(\pi\). The fourth condition restricts the steady-state solution to only the non-cooperative Nash equilibria.

**Proposition 3:**

If 
\[
\left( \frac{1}{(1-\pi)\pi R} \right) \leq \frac{k_0}{k_1} \leq \left( \frac{3(1-\pi)\theta}{(1-\theta)} \right),
\]

then  
\[
A = 0, \quad B = \frac{(1-\theta)\pi^{(1-\theta)} R}{3}
\]

\[
\lambda_1 = \sqrt{B}, \quad \lambda_2 = -\sqrt{B}
\]

\[
\mu_{ss} = B - 1
\]

The first inequality implies that \(r_0^* \leq R\rho\), while the second inequality implies that \(r_1^* \leq R\rho\).

Appendix C proves that these two inequalities are both necessary and sufficient for steady-state growth. In every period, a new banking coalition forms, accepts deposits, and allocates \((1-\pi)\) to consumer lending and \(\pi\) to capital. Therefore, capital accumulates at the rate \(B - 1\) in every period. As before, there can be either negative steady-state growth \((B < 1)\) or positive steady-state growth \((B > 1)\).

As in proposition 1, the economy under proposition 3 is stable since there are two real roots of equal absolute value. If \(B < 1\), then the stock of capital and output converge to zero. If \(B > 1\), then the two variables grow at the positive rate of \(B - 1\) through time. Along the steady-state growth path, the two-period deposit rate \(r_t^2\) remains constant, while the one-period
deposit rate and consumer loan rate fluctuate between odd and even years. Moreover, as long as \( \bar{k}_0 \) is not large relative to \( \bar{k}_1 \) and \( \pi \) is not close to one, proposition is likely to hold.

In this model, financial intermediation increases the steady-state growth rate and eliminates the volatility in investment and output. Appendix C shows that \( \mu_{ss} \) under proposition 3 exceeds \( \mu_{ss} \) under proposition 1. As in Bencivenga and Smith (1991) and Levine (1991), banks increase the steady-state growth rate by allocating a higher percentage of wealth into capital and by preventing the premature liquidation of capital. Furthermore, banks in this model decrease the volatility in output by allocating the same percentage of wealth into capital and into consumer lending. With no aggregate uncertainty, competition in the banking sector forces each bank to choose the same utility-maximizing allocation of capital and lending which eliminates the volatility in asset rates of return. As a result, investment rates in capital remain constant through time which keeps capital and output on their steady-state growth paths. Financial intermediation reduces volatility in per capita output not through diversification of production risk as in Greenwood and Jovanovic (1990) and Acemoglu and Zilibotti (1997), but rather by eliminating the volatility in rates of return.  

6. CONCLUSION

This paper shows that the volatility in per capita growth rates may be explained by agents’ responses to uncertainty under different financial structures. Under a poorly-developed financial sector, agents must cover their consumption risk by investing directly in two assets: a one-period consumer loan and a two-period capital good. Due to incomplete markets and incomplete participation in those markets, the asset rates of return are not arbitraged together. As a result, the economy experiences wide fluctuations in the levels of net investment and output. Under a well-developed financial sector, agents can cover their consumption risk by

\[9\text{ Mention work by Allen and Gale}\]
depositing their savings in a bank. Competition in the banking sector forces each bank to make
the same portfolio decision and thus post the same set of deposit rates in every time period.
Therefore, the mean levels of net investment and output are higher, while the volatility is lower
under a well-developed financial system. This result conforms with the empirical evidence that
shows a negative connection between the initial level of financial development and the
volatility in per capita growth rates.
REFERENCES


Penn World Table (Mark 5.6a), National Bureau of Economic Research (1995).


APPENDIX A

1. The Return on Capital:

The total profit or total return on capital of each entrepreneur at time \( t + 3 \) is

\[
\Pi_{t+3} = y_{t+3} - l_{1,t+2}w_{1,t+2} = \bar{k}^{\delta} k^{\delta} l_{t+3}^{1-\theta} - l_{1,t+2}w_{1,t+2}
\]

(A.1)

where the production function (5) was used in the second equality. Each entrepreneur selects \( l_{1,t+2} \) to maximize (A.1). This yields the market-clearing wage (7).

Substituting in (7) for \( w_{1,t+2} \) and the per entrepreneur labor supply, \( 1/\pi \), for \( l_{1,t+2} \), the total return on capital of each entrepreneur becomes

\[
\Pi_{t+3} = \bar{k}_{t+3}(1/\pi)^{1-\theta} - \bar{k}_{t+3}(1-\theta)\pi^{\theta-1} = \bar{k}_{t+3}\theta\pi^{\theta-1}.
\]

(A.2)

Dividing each side by \( \bar{k}_{t+3} \), one obtains equation (8) of the text.

2. Commodity Market Equilibrium:

Let \( \{ e^{t}, f^{t}, g^{t}, h^{t} \} \) represent the percentage of labor income \( w_{t} \) an agent born at \( t \) devotes to capital, storage, consumer lending and consumer borrowing, respectively. Therefore, at any time \( t \), the total supply of the consumption good equals the sum of total production by the \( \pi \) firms and the return to storage:

\[
Y_{t} = \pi y_{t} + w_{1,t-2}f^{t-2}n.
\]

(A.3)

Total spending, on the other hand, equals consumption by the four overlapping generations plus investment into capital and storage:

\[
Y_{t} = C_{0,t} + C_{1,t-1} + C_{2,t-2} + C_{3,t-3} + w_{1,t-1}e^{t-1} + w_{1,t-1}f^{t-1}.
\]

(A.4)

Set (A.3) equal to (A.4) and substitute in the levels of consumption from (12):

\[
\pi y_{t} + w_{1,t-2}f^{t-2}n + = w_{1,t}h^{t} + w_{1,t-1}(1-e^{t-1} - f^{t-1} - g^{t-1} - h^{t-1}r^{t}) + w_{1,t-2}(f^{t-2}n + g^{t-2}r^{t}) + w_{1,t-3}(\pi e^{t-3} R \rho) + w_{1,t-1}e^{t-1} + w_{1,t-1}f^{t-1}
\]
Canceling out common terms, the above condition reduces to

$$\pi y_t = w_{1,t} h^t + w_{1,t-1}(1 - g^{l-1} - h^{l-1} r^t) + w_{1,t-2}(g^{l-2} r^t) + w_{1,t-3}(\pi e^{l-3} R\rho). \quad (A.5)$$

Substituting in the consumer loan equilibrium conditions (3) and (4) into (A.5), one obtains

$$\pi y_t = w_{1,t-1} + w_{1,t-3}(\pi e^{l-3} R\rho). \quad (A.6)$$

Plugging in the production function (5) and then the per firm labor supply $1/\pi$, the left-hand side of (A.6) equals

$$\pi y_t = \pi k_i \bar{\pi}^{1-\theta} \theta_{i,j}^{1-\theta} = \pi k^*_i (1/\pi) = \pi^0 k_i. \quad (A.7)$$

Using equation (7) for $w_{1,t-1}$, (8) for $\rho$ and (11) for $w_{1,t-3}(\pi e^{l-3} R)$, the right-hand side of (A.6) equals

$$w_{1,t-1} + w_{1,t-3}(\pi e^{l-3} R\rho) = \bar{k}_i (1-\theta) \pi^0 + \pi k_i \rho = \bar{k}_i (1-\theta) \pi^0 + \pi k_i (\theta \pi^{\omega_1})$$

$$= \pi^0 \bar{k}_i. \quad (A.8)$$

Q.E.D.

APPENDIX B

1. Consumer Optimization:

Solving the first-order conditions (15) - (18), one derives the optimal holdings of assets under the three solution types:

$$e^t = \frac{\pi R\rho - r^t_{i,1}}{3(R\rho - r^t_{i,1})} \quad \text{for corner \\ & interior} \quad (r^t_{i+1} \leq \pi R\rho) \quad (A.9)$$

$$= 0 \quad \text{for stagnation} \quad \quad (r^t_{i+1} > \pi R\rho)$$

$$f^t = \frac{R\rho(1-\pi)}{3(R\rho - n)} - \frac{w_{1,i+1}}{w_{1,i}(3n)} \quad \text{for corner} \quad \quad (r^t_{i+1} \leq n) \quad (A.10)$$

$$= 0 \quad \text{for interior \\ & stagnation} \quad (r^t_{i+1} > n)$$
\[
g'_t = \frac{w_{t+1}}{w_t (3n)} \quad \text{for corner} \quad \quad (r^{*}_{t+1} \leq n) \quad \text{(A.11)}
\]

\[
= \frac{R \rho (1 - \pi)}{3(R \rho - r^*_t)} \quad \text{for interior} \quad \quad (\pi R \rho \geq r^{*}_{t+1} > n)
\]

\[
= \frac{1}{3} \quad \text{for stagnation} \quad \quad (r^{*}_{t+1} > \pi R \rho)
\]

\[
h^{t+1} = \frac{1}{3r^{*}_{t+1}} \quad \text{for all solutions} \quad \quad (r^{*}_{t+1} > 0) \quad \text{(A.12)}
\]

Setting (A.11) equal to (A.12), the market-clearing consumer loan rate at time \(t + 1\) is:

\[
r^{*}_{t+1} = \frac{R \theta \pi^{\alpha -1}}{\bar{k}_{t+1}} \quad \text{for corner & interior} \quad \quad (r^{*}_{t+1} \leq \pi R \rho) \quad \text{(A.13)}
\]

\[
= \frac{\bar{k}_{t+2}}{\bar{k}_{t+1}} \quad \text{for stagnation} \quad \quad (r^{*}_{t+1} > \pi R \rho).
\]

Notice that if capital accumulates at a constant two-period rate then \(r^{*}_{t+1}\) cycles between one value for odd \(t\) and another value for even \(t\).

2. General Solution:

Before proving proposition 1 and 2, we describe the conditions that must be met at any time \(t\) for a general solution:

\[
\bar{k}_t = a(\lambda_1)^t + b(\lambda_2)^t \quad \text{(A.14)}
\]

\[
\bar{k}_0 = a + b \quad \text{(A.15)}
\]

\[
\bar{k}_1 = a(\lambda_1) + b(\lambda_2) \quad \text{(A.16)}
\]

where \(\lambda_1\) and \(\lambda_2\) represent the positive and negative roots of (19) and where \(a\) and \(b\) are any numbers that satisfy our initial conditions. See Bamoul (1970), pg. 173 for details. Equations (A.14) - (A.16) can be combined to produce
\[
\overline{k}_t = \left[ \frac{\lambda_2 \tilde{k}_0 - \tilde{k}_1}{\lambda_2 - \lambda_1} \right] (\lambda_1)' + \left[ \frac{\tilde{k}_1 - \lambda_1 \tilde{k}_0}{\lambda_2 - \lambda_1} \right] (\lambda_2)'.
\] (A.17)

for all \( t \geq 0 \). Since \( \lambda_1 > 0 \) and \( \lambda_2 < 0 \), then \( a > 0, b < 0 \) and \(|a| > |b|\).

3. Proof of Proposition 1:

We first derive (A.17) assuming that proposition 1 holds (sequence of corner solutions) and then we derive the necessary and sufficient conditions for proposition 1.

Replace \( r'_{n+1} \) in (A.9) with \( n \). Next, substitute (A.9) and the wage rate (7) into (11)

\[
\overline{k}_{t+3} = \left[ \frac{1 - \theta}{\pi} (\theta \pi^\theta R - n) R \right] \left[ \frac{3 (\theta \pi^\theta R - n)}{3 (\theta \pi^\theta R - n)} \right] \overline{k}_{t+1}.
\] (A.18)

where \( \lambda_1 = \left| \lambda_2 \right| \). Using (A.17), the general solution under proposition 1 is

\[
\overline{k}_t = \left[ \frac{\lambda_2 \tilde{k}_0 + \tilde{k}_1}{2 \lambda_1} \right] (\lambda_1)' + \left[ \frac{\lambda_1 \tilde{k}_0 - \tilde{k}_1}{2 \lambda_1} \right] (-\lambda_1)'.
\] (A.19)

for all \( t \geq 0 \). Since the coefficient in front of the positive root \( \lambda_1 \) is greater in value, \( \lambda_1 \) eventually dominates and thus the system will not oscillate. Therefore, \( \overline{k}_t = \tilde{k}_0 (\lambda_1)' \) for even \( t \) and \( \overline{k}_t = \tilde{k}_1 (\lambda_1)^{-1} \) for odd \( t \) where the two-period growth rate \( \mu_t \) equals \( B - 1 \).

The necessary conditions for proposition 1 is that the economy begins the first two periods at the corner solution: \( r_0^* \leq n \) and \( r_1^* \leq n \). Using (A.13), the condition \( r_0^* \leq n \) holds if

\[
\left( \frac{\overline{k}_1 R \theta \pi^{\theta-1}}{\tilde{k}_0 R \theta \pi^{\theta-1} (1 - \pi) + \tilde{k}_1} \right) \leq n.
\] (A.20)

Using (A.13), the condition \( r_1^* \leq n \) holds if

\[
\left( \frac{\tilde{k}_0 (1 - \theta) R \theta \pi^{\theta-1} (R \theta \pi^\theta - n)}{3 \tilde{k}_1 \theta \pi^{-1} (1 - \pi)(R \theta \pi^{\theta-1} - n) + \tilde{k}_0 (1 - \theta) (R \theta \pi^\theta - n)} \right) \leq n
\] (A.21)
where \( \mu_t \) in (A.18) was used to substitute out \( \vec{k}_2 \). Conditions (A.20) and (A.21) can be rewritten as the inequality in proposition 1. If both (A.20) and (A.21) hold, then capital accumulates at the two-period rate of \( B - 1 \) at time=0 and 1. As a result, the market-clearing \( r_t^* \) equals the left-hand side of (A.20) for even periods and the left-hand side of (A.21) for odd periods. Thus, the economy moves from corner solution to corner solution where \( \mu_t = B - 1 \) for all \( t \geq 2 \). Therefore, conditions (A.20) and (A.21) are also sufficient for steady-state growth of \( B - 1 \).

4. Proof of Proposition 2:

We first show that along the stable arm the economy follows a steady-state growth path. We then show that along any of the unstable arms the economy does not converge to the steady-state growth path.

Assuming \( \pi R \rho \geq r^*_{t+1} > n \), substitute (A.13) into (A.9) to get a value for \( \epsilon' \). Next, substitute \( \epsilon' \) and the wage rate (7) into (11)

\[
\vec{k}_{t+3} = -\left\{ \frac{(1-\theta)\pi}{3\theta} \right\} \vec{k}_{t+2} + \left\{ \frac{(1-\theta)\pi^{\alpha+1}R}{3} \right\} \vec{k}_{t+1}.
\]  

(A.22)

where \( |\lambda_2| > |\lambda_1| \). If we start the economy on the stable arm (\( \vec{k}_t = \lambda_i \vec{k}_0 \)), (A.17) reduces to \( \vec{k}_t = \vec{k}_0 (\lambda_i)^t \). As a result, the two-period growth rate \( (\lambda_i)^2 - 1 \) is constant across all \( t \).

According to (A.13), \( r^*_{t+1} \) is also constant for all \( t \). Therefore, since \( r^*_{t+1} < \pi R \rho \) by assumption (10), the necessary condition for steady-state growth is that \( r^*_{t+1} > n \). The sufficient condition is that when \( \mu_t = (\lambda_i)^2 - 1 \) the economy remain in the interior solution. This condition is

\[
n < r^*_{t+1} < \pi R \rho \quad \text{or condition (ii) of proposition 2.}
\]

Along any unstable branch, the stock of capital eventually oscillates around zero. If \( |\lambda_2| < 1 \), then the oscillations dampen. If \( |\lambda_2| > 1 \), then the oscillations explode. However, the economy is unstable in either case since \( \vec{k}_t < 0 \) for finite \( t \). Recall that (A.22) describes the evolution of capital assuming the economy remains in the interior solution. According to (A.9), investment at time \( t + 1 \) and thus the capital stock at time \( t + 3 \) falls below zero when \( r^*_{t+1} \) rises above \( \pi R \rho \). When this occurs, the economy enters the stagnation solution.
APPENDIX C

1. Proof of Proposition 3:

First, we demonstrate that \( \bar{k}_0 / \bar{k}_1 \geq [(1 - \pi)\pi (R \theta \pi^\theta)]^{-1} \) satisfies the necessary condition for the steady-state growth path with the pair \( \{ q', d^{-1} \} = \{ \pi, 1/3 \} \). Second, we show that \( \bar{k}_0 / \bar{k}_1 \leq 3(1 - \pi)\theta / (1 - \theta) \) is the sufficient condition for the steady-state. Lastly, we find that \( \{ q', d^{-1} \} = \{ \pi, 1/3 \} \) is the only pair of values that satisfies (i)-(iv) of definition 2.

Assuming conditions (i)-(iv) of definition 2 hold, substitute \( q'^{+1} = \pi \) and \( \alpha_{t+3} = 1 \) into (11)

\[
\bar{k}_{t+3} = \left[ \frac{(1 - \theta)\pi^{\theta-1} R}{3} \right] \bar{k}_{t+1} \tag{A.23}
\]

where \( \lambda_i = |\lambda_i| \). Therefore, as in proposition 1, \( \bar{k}_t = \bar{k}_0 (\lambda_t)^t \) for even \( t \) and \( \bar{k}_t = \bar{k}_1 (\lambda_t)^{-t} \) for odd \( t \) where the two-period growth rate \( \mu_t \) equals \( B - 1 \). Since the banks supply \( w_{t,j-1} (1 - \pi) / 3 \) in loans at time \( t \), \( r^* = \bar{k}_{t+1} / \bar{k}_t (1 - \pi) \) along the “intermediated” growth path.

If the banking coalition sets \( q^0 \) equals to \( \pi \), then the ex ante utility of the representative depositor is maximized. As a result, no other coalition can do better and thus (iv) holds. To honor its posted deposit rates, the coalition at time=0 must induce the early consumers to withdraw after one period and the late consumers to withdraw after two periods. In other words, the self-selection constraints (26) and (27) must hold at time=0. When \( q^0 = \pi, r^0_l \) equals \( r^0_0 \) and \( r^0_s \) equals \( R \theta \). For constraint (26) to hold, \( r^0_0 = r^0_1 = \frac{\bar{k}_1}{2} \bar{k}_0 (1 - \pi) \leq R \theta \). This is the first inequality in proposition 3. Moreover, since \( r^0_1 = r^0_0 \), (27) binds and agents deposit not for arbitrage purposes, but to save for age-2 and age-3 consumption. Therefore, (ii) and (iii) hold.

Each bank can guarantee a return of \( r^0_l \) to the early consumers and a return of \( R \theta \) to the late consumers. Recall that under financial autarky early consumers receive a return less than \( r^0_t \) due to the premature liquidation of capital and late consumers receive a return less than \( R \theta \) due to positive consumer lending. Since the age-1 agents receive a higher ex ante return, all savings is deposited and thus (iv) holds.

Sufficiency requires that the market-clearing loan rate \( r^* \) remain below \( R \theta \) so that self-segregation of depositors can continue. The first inequality \( \bar{k}_0 / \bar{k}_1 \geq [(1 - \pi)\pi (R \theta \pi^\theta)]^{-1} \) ensures
that \( r_t^* < R \rho \) for even \( t \). Substituting in (A.23) into the expression for \( r_t^* \), the second inequality \( \bar{k}_0 / \bar{k}_1 \leq 3(1 - \pi)\theta / (1 - \theta) \) ensures that \( r_t^* < R \rho \) for even \( t \).

What happens, however, if either \( r_0^* \) or \( r_1^* \) is strictly greater than \( R \rho \)? To induce self-selection, the banking coalition could increase \( q' \) \((t=0,1)\) by say \( \varepsilon \) to raise \( r_t^2 \) to \( r_t^1 \). As a result, however, consumer lending dominates deposits. This prompts the age-1 agents to lend all of their savings directly to the age-0 generation which would violate (i) of definition 2. Therefore, if both inequalities did not hold, then conditions (i)-(iv) cannot be met and thus the economy remains in autarky.

2. **Comparison of Steady-State Growth Rates:**

The following shows that the steady-state growth rate under proposition 3 exceeds that the growth rate under proposition 1:

\[
\left[ \frac{(1 - \theta)\pi^{\alpha-1} R}{3} \right] > \left[ \frac{(1 - \theta)\pi^{\alpha}(\theta\pi^\theta R - n)R}{3(\theta\pi^{\alpha-1} R - n)} \right] \tag{A.24}
\]

This can be reduced to

\[
\theta\pi^{\alpha-1} R(1 - \pi^2) > (1 - \pi)n \tag{A.25}
\]

Since \( \theta\pi^{\alpha-1} R > n \) by assumption (9), then the growth under proposition 3 exceeds growth under proposition 1 except if \( \pi = 1 \) (no liquidity risk case).

We are unable to show that the steady-state growth rate under proposition 3 exceeds that the growth rate under proposition 2 due to nonlinearities.
Figure 1
Cross-Decade Correlations of Financial Measures
Figure 2
Volatility of Per Capita Growth
Figure 3 -- The Consumer Loan Market at time $t + 1$